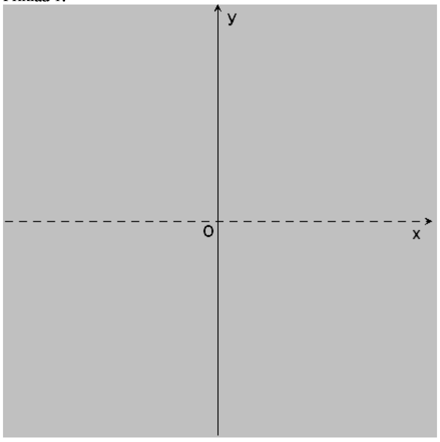
1. $f(x,y) = \frac{x}{y}$.

$$y \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \neq 0\}.$$

Příklad 1:

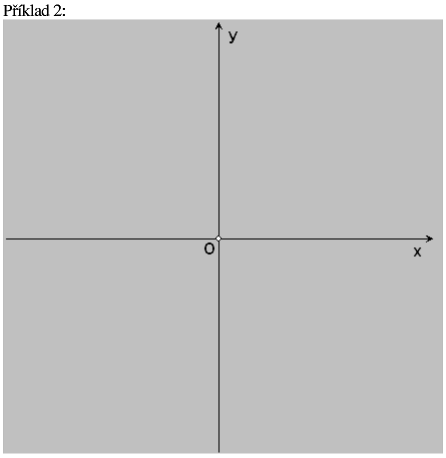


2. $f(x,y) = \frac{3}{x^2+y^2}$.

$$x^2 + y^2 \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; (x,y) \neq (0,0)\}.$$

Příklad 2:



3. $f(x,y) = \frac{x^2+y}{x^2-y^2}$.

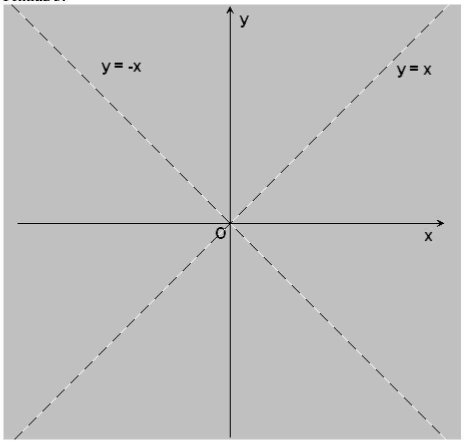
$$x^2 - y^2 \neq 0$$

$$(x-y) \cdot (x+y) \neq 0$$

$$x \neq y \,\wedge\, x \neq -y.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x \neq y \wedge x \neq -y\}.$$

Příklad 3:

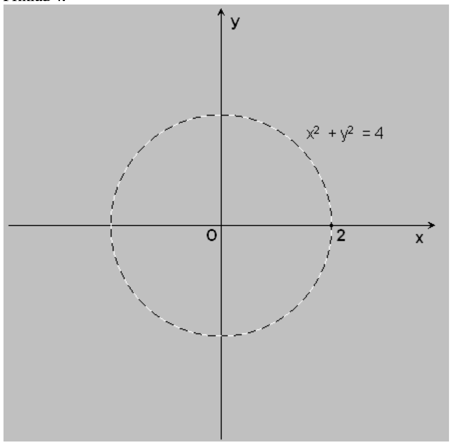


4. $f(x,y) = \frac{xy}{4-x^2-y^2}$.

$$4 - x^2 - y^2 \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 \neq 4\}.$$

Příklad 4:

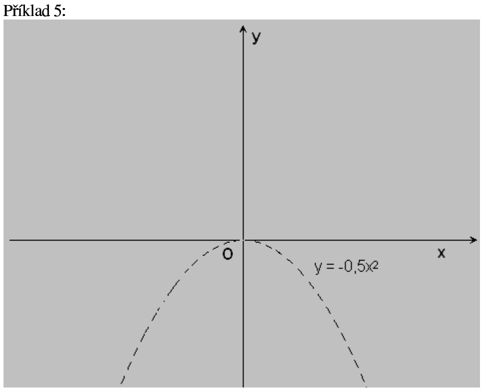


5. $f(x,y) = \frac{x^2-1}{x^2+2y}$.

$$x^2 + 2y \neq 0 \quad \Rightarrow \quad y \neq -\frac{1}{2}x^2.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \neq -\frac{1}{2}x^2\}.$$

Příklad 5:



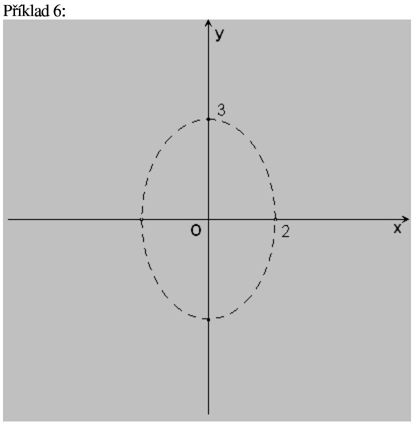
6. $f(x,y) = \frac{x+y}{9x^2+4y^2-36}$.

$$9x^2 + 4y^2 - 36 \neq 0 \Rightarrow 9x^2 + 4y^2 \neq 36$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} \neq 1.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; 9x^2 + 4y^2 \neq 36\}.$$

Příklad 6:

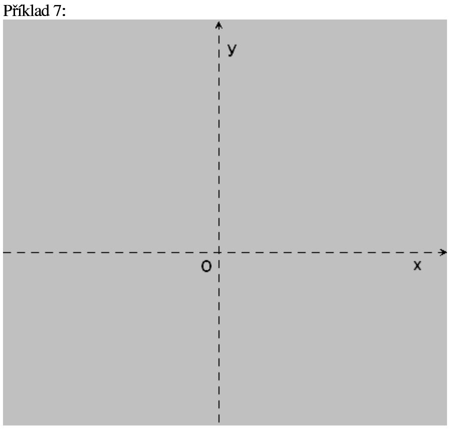


7. $f(x,y) = \frac{1}{xy}$.

$$xy \neq 0 \quad \Rightarrow \quad x \neq 0 \vee y \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x \neq 0 \vee y \neq 0\}.$$

Příklad 7:



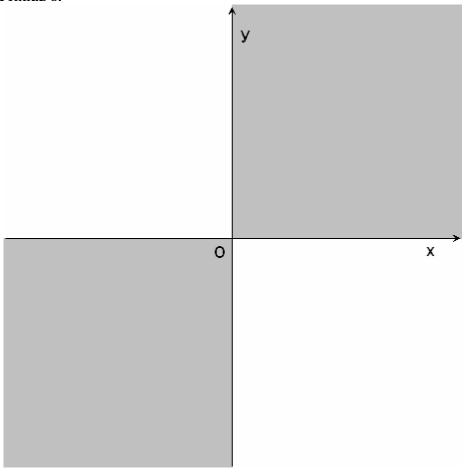
8. $f(x,y) = \sqrt{xy}$.

$$xy \geq 0$$

$$(x \geq 0 \wedge y \geq 0) \quad \vee \quad (x \leq 0 \wedge y \leq 0).$$

$$D_f = \{(x,y) \in \mathbb{R}^2; (x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0)\}.$$

Příklad 8:

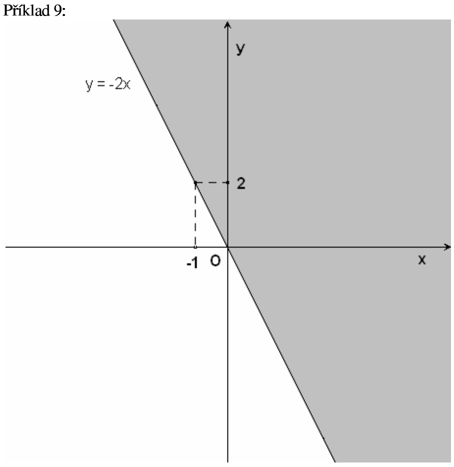


9. $f(x,y) = \sqrt{2x+y}$.

$$2x + y \geq 0 \Rightarrow y \geq -2x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \geq -2x\}.$$

Příklad 9:

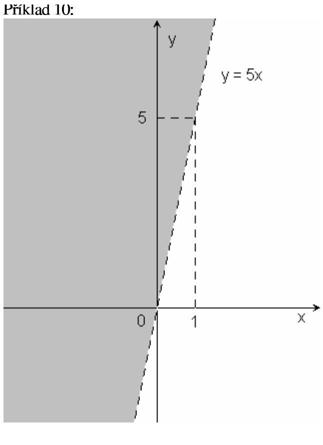


10. $f(x,y) = \frac{x}{\sqrt{y-5x}}$.

$$y - 5x > 0 \Rightarrow y > 5x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y > 5x\}.$$

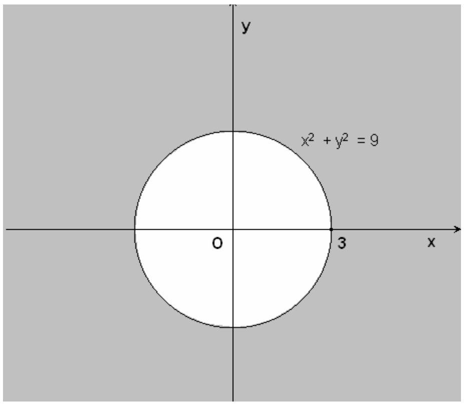
Příklad 10:



11. $f(x,y) = \sqrt{x^2 + y^2 - 9}$.

$$x^2 + y^2 - 9 \geq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 \geq 9\}.$$



12. $f(x,y) = \sqrt{16 - (x^2 + y)^2}$.

$$16 - \left(x^2 + y\right)^2 \geq 0$$

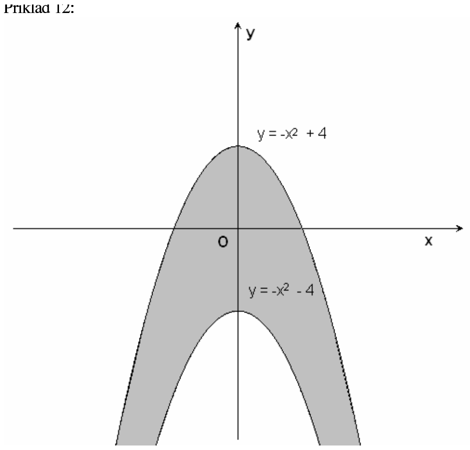
Odtud

$$\left(x^2 + y\right)^2 \leq 16 \quad \Rightarrow \quad -4 \leq x^2 + y \leq 4$$

$$\Rightarrow \quad -4 - x^2 \leq y \leq 4 - x^2.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; -4 - x^2 \leq y \leq 4 - x^2\}.$$

Příklad 12:



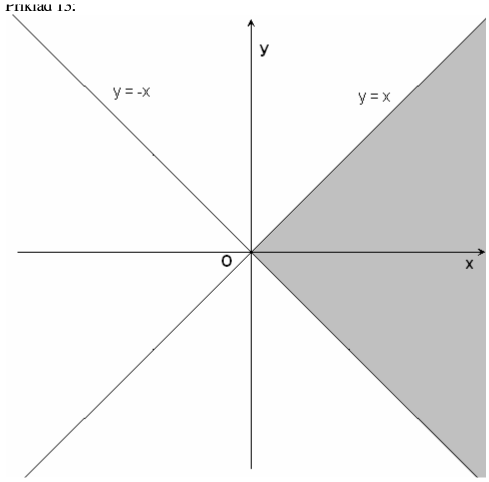
13. $f(x) = 2\sqrt{x+y} - 5\sqrt{x-y}$.

$$x + y \geq 0 \ \wedge \ x - y \geq 0$$

$$y \geq -x \ \wedge \ y \leq x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; -x \leq y \leq x\}.$$

Příklad 13:



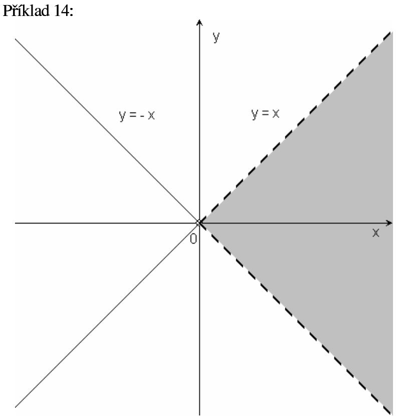
14. $f(x,y) = \frac{1}{\sqrt{x+y}} + \frac{1}{\sqrt{x-y}}$.

$$x + y > 0 \ \wedge \ x - y > 0$$

$$y > -x \ \wedge \ y < x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; -x < y < x\}.$$

Příklad 14:



15. $f(x,y)=\sqrt{4-x^2}+\sqrt{1-y^2}$.

$$4-x^2\geq 0 \quad \wedge \quad 1-y^2\geq 0$$

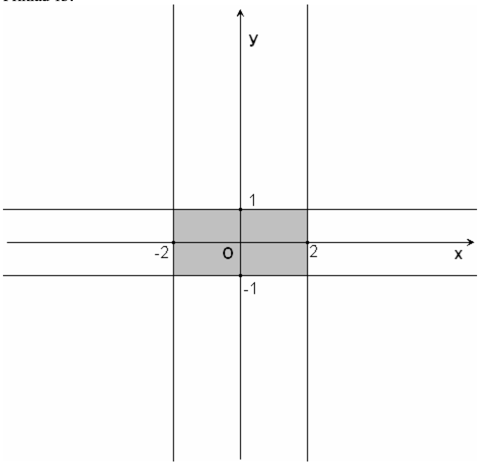
$$x^2\leq 4 \quad \wedge \quad y^2\leq 1$$

$$|x|\leq 2 \quad \wedge \quad |y|\leq 1$$

$$-2\leq x\leq 2 \quad \wedge \quad -1\leq y\leq 1$$

$$D_f=\{(x,y)\in\mathbb{R}^2; -2\leq x\leq 2 \wedge -1\leq y\leq 1\}.$$

Příklad 15:

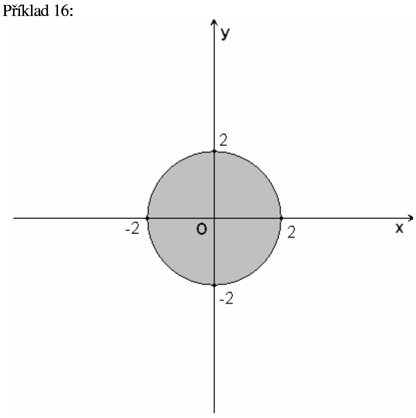


16. $f(x,y)=\left(x^2+y^2\right)\sqrt{4-x^2-y^2}$.

$$4-x^2-y^2\geq 0 \quad \Rightarrow \quad x^2+y^2\leq 4.$$

$$D_f=\{(x,y)\in\mathbb{R}^2; x^2+y^2\leq 4\}.$$

Příklad 16:

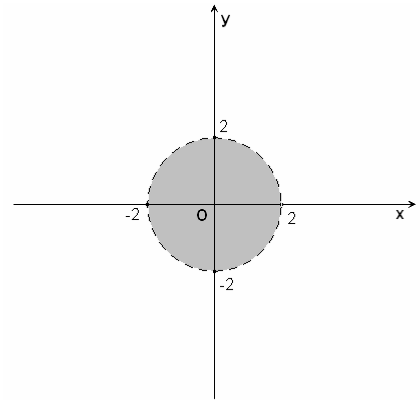


17. $f(x,y)=\frac{x^2+y^2}{\sqrt{4-x^2-y^2}}$.

$$4-x^2-y^2>0 \quad \Rightarrow \quad x^2+y^2<4.$$

$$D_f=\{(x,y)\in\mathbb{R}^2; x^2+y^2<4\}.$$

Příklad 17:

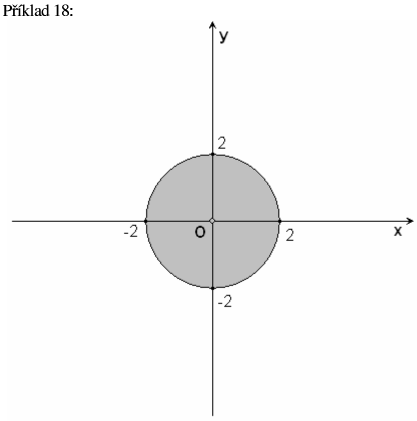


18. $f(x,y)=\frac{\sqrt{4-x^2-y^2}}{x^2+y^2}$.

$$4-x^2-y^2\geq 0 \quad \wedge \quad x^2+y^2\neq 0.$$

$$D_f=\{(x,y)\in\mathbb{R}^2; x^2+y^2\leq 4 \wedge (x,y)\neq(0,0)\}.$$

Příklad 18:



19. $f(x,y) = \sqrt{(x^2+y^2-25)\,(16-x^2-y^2)}$.

$$\left(x^2+y^2-25\right)\cdot\left(16-x^2-y^2\right) \geq 0.$$

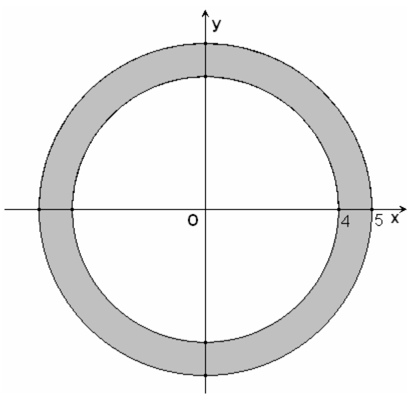
$$\left(x^2+y^2-25 \geq 0 \wedge 16-x^2-y^2 \geq 0\right) \quad \vee \quad \left(x^2+y^2-25 \leq 0 \wedge 16-x^2-y^2 \leq 0\right)$$

$$\left(x^2+y^2 \geq 25 \quad \wedge x^2+y^2 \leq 16\right) \quad \vee \quad \left(x^2+y^2 \leq 25 \quad \wedge x^2+y^2 \geq 16\right)$$

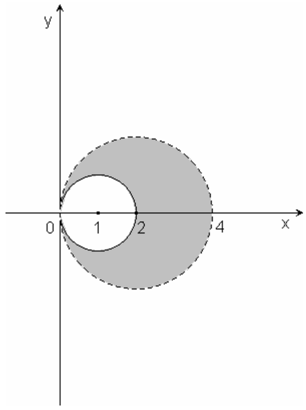
$$(x,y) \in \emptyset \qquad \vee \qquad 16 \leq x^2+y^2 \leq 25.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; 16 \leq x^2+y^2 \leq 25\}.$$

Příklad 19:



Příklad 20:



20. $f(x,y) = \sqrt{\frac{x^2-2x+y^2}{4x-x^2-y^2}}$.

$$\frac{x^2-2x+y^2}{4x-x^2-y^2} \geq 0.$$

$$\left(x^2-2x+y^2 \geq 0 \wedge 4x-x^2-y^2 > 0\right) \quad \vee \quad \left(x^2-2x+y^2 \leq 0 \wedge 4x-x^2-y^2 < 0\right).$$

Protože

$$\begin{aligned} x^2-2x+y^2 &= x^2-2x+1-1+y^2 = (x-1)^2+y^2-1 \\ 4x-x^2-y^2 &= -\left(x^2-4x+y^2\right) = -\left(x^2-4x+4-4+y^2\right) = -\left[(x-2)^2+y^2-4\right] \\ &= -(x-2)^2-y^2+4, \end{aligned}$$

máme

$$\left[(x-1)^2+y^2-1 \geq 0 \wedge -(x-2)^2-y^2+4 > 0\right] \vee \left[(x-1)^2+y^2-1 \leq 0 \wedge -(x-2)^2-y^2+4 < 0\right]$$

$$\left[(x-1)^2+y^2 \geq 1 \wedge (x-2)^2+y^2 < 4\right] \quad \vee \quad \left[(x-1)^2+y^2 \leq 1 \wedge (x-2)^2+y^2 > 4\right]$$

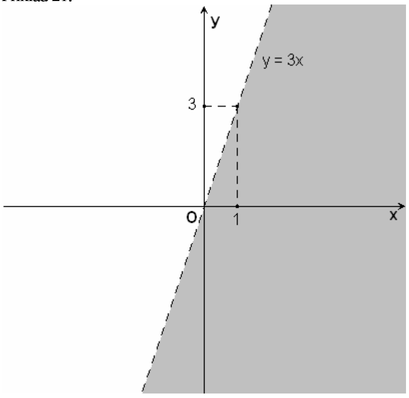
$$D_f = \{(x,y) \in \mathbb{R}^2; (x-1)^2+y^2 \geq 1 \wedge (x-2)^2+y^2 < 4\}.$$

21. $f(x,y) = \ln(3x - y)$.

$$3x - y > 0 \quad \Rightarrow \quad y < 3x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y < 3x\}.$$

Příklad 21:

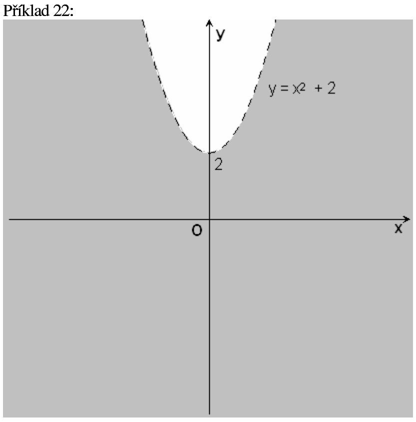


22. $f(x,y) = \log\left(x^2 - y + 2\right)$.

$$x^2 - y + 2 > 0 \quad \Rightarrow \quad y < x^2 + 2.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y < x^2 + 2\}.$$

Příklad 22:

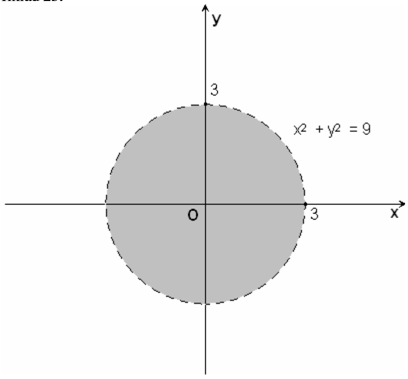


23. $f(x,y) = \ln\left(9 - x^2 - y^2\right)$.

$$9 - x^2 - y^2 > 0 \quad \Rightarrow \quad x^2 + y^2 < 9.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 < 9\}.$$

Příklad 23:



24. $f(x,y) = x - \ln\left(y + \frac{1}{x}\right)$.

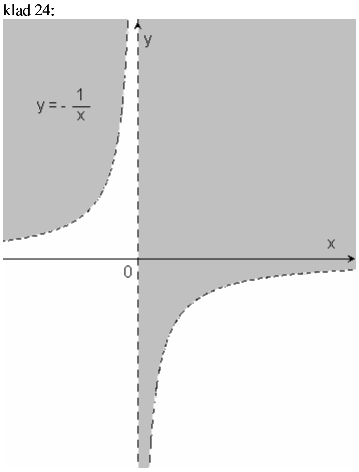
$$y + \frac{1}{x} > 0 \quad \wedge \quad x \neq 0$$

$$\frac{xy+1}{x} > 0 \quad \wedge \quad x \neq 0$$

$$[(xy + 1 > 0 \wedge x > 0) \vee (xy + 1 < 0 \wedge x < 0)] \wedge x \neq 0$$

$$\left[\left(y > -\frac{1}{x} \wedge x > 0\right) \vee \left(y > -\frac{1}{x} \wedge x < 0\right)\right] \wedge x \neq 0$$

klad 24:



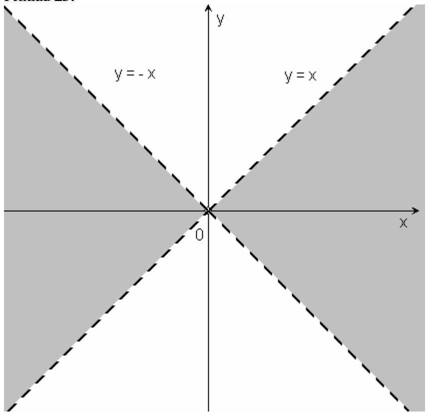
25. $f(x,y) = \ln \frac{x+y}{x-y}$.

$$\frac{x+y}{x-y} > 0$$

$$(x+y>0 \wedge x-y>0) \quad \vee \quad (x+y<0 \wedge x-y<0)$$

$$(y>-x \wedge y<x) \quad \vee \quad (y<-x \wedge y>x)$$

Příklad 25:



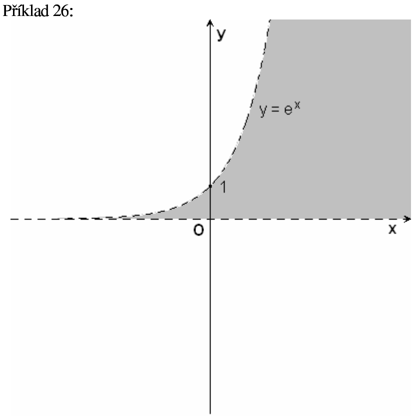
26. $f(x,y) = \ln(x - \ln y)$.

$$x - \ln y > 0 \quad \wedge \quad y > 0$$

$x - \ln y > 0 \Rightarrow \ln y < x \Rightarrow \ln y < \ln e^x \Rightarrow y < e^x$.

$$D_f = \{(x,y) \in \mathbb{R}^2; y < e^x \wedge y > 0\}.$$

Příklad 26:



27. $f(x,y) = \frac{x+y}{\ln(1-x^2-y^2)}$.

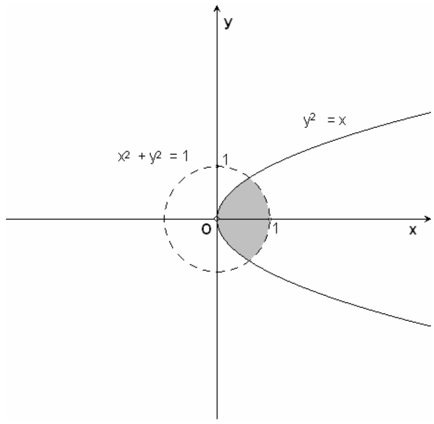
$$\ln\left(1-x^2-y^2\right) \neq 0 \quad \wedge \quad 1-x^2-y^2 > 0$$

$$1-x^2-y^2 \neq 1 \quad \wedge \quad x^2+y^2 < 1$$

$$x^2+y^2 \neq 0 \quad \wedge \quad x^2+y^2 < 1.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x^2+y^2 < 1 \wedge (x,y) \neq (0,0)\}.$$

Příklad 28:



28. $f(x,y) = \frac{\sqrt{x-y^2}}{\ln(1-x^2-y^2)}$.

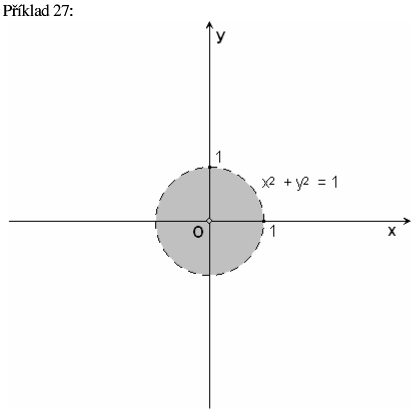
$$x-y^2 \geq 0 \quad \wedge \quad \ln\left(1-x^2-y^2\right) \neq 0 \quad \wedge \quad 1-x^2-y^2 > 0$$

Odtud

$$x \geq y^2 \quad \wedge \quad x^2+y^2 \neq 0 \quad \wedge \quad x^2+y^2 < 1.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x \geq y^2 \wedge x^2+y^2 < 1 \wedge (x,y) \neq (0,0)\}.$$

Příklad 27:



29. $f(x,y) = \ln\left[y \cdot \ln(x-y)\right].$

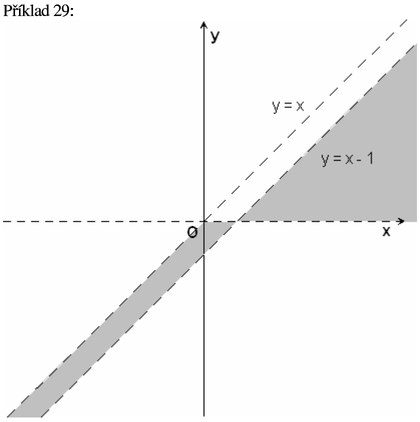
$$y \cdot \ln(x-y) > 0 \quad \wedge \quad x - y > 0$$

1)

$$\begin{aligned} y \cdot \ln(x-y) > 0 \quad &\Rightarrow \quad [y > 0 \wedge \ln(x-y) > 0] \quad \vee \quad [y < 0 \wedge \ln(x-y) < 0] \\ &\Rightarrow \quad (y > 0 \wedge x - y > 1) \quad \vee \quad (y < 0 \wedge x - y < 1) \\ &\Rightarrow \quad (y > 0 \wedge y < x - 1) \quad \vee \quad (y < 0 \wedge y > x - 1). \end{aligned}$$

2) $x - y > 0 \quad \Rightarrow \quad y < x.$

Příklad 29:

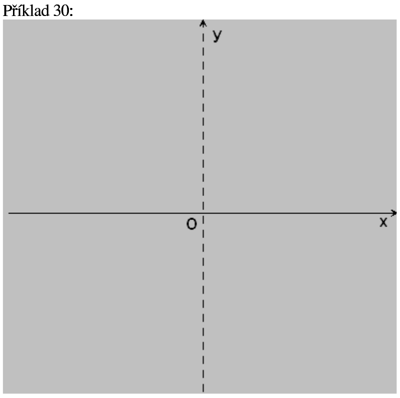


30. $f(x,y) = \sin \frac{y}{x}.$

$$x \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x \neq 0\}.$$

Příklad 30:

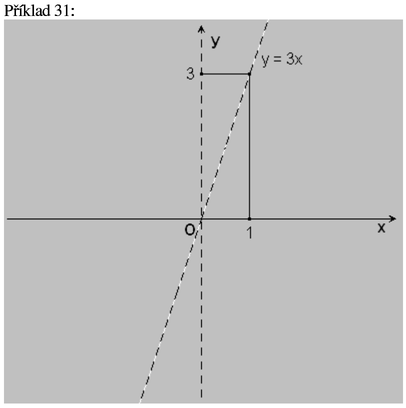


31. $f(x,y) = \frac{1}{x} + \cos \frac{1}{3x-y}.$

$$x \neq 0 \wedge 3x - y \neq 0 \quad \Rightarrow \quad x \neq 0 \wedge y \neq 3x.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; x \neq 0 \wedge y \neq 3x\}.$$

Příklad 31:

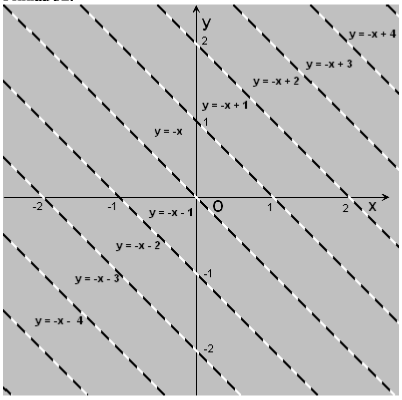


32. $f(x,y) = \cot \pi (x+y)$.

$$\pi (x+y) \neq k\pi,\ k \in \mathbb{Z} \quad \Rightarrow \quad x+y \neq k \quad \Rightarrow \quad y \neq -x+k.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \neq -x+k,\ k \in \mathbb{Z}\}.$$

Příklad 32:

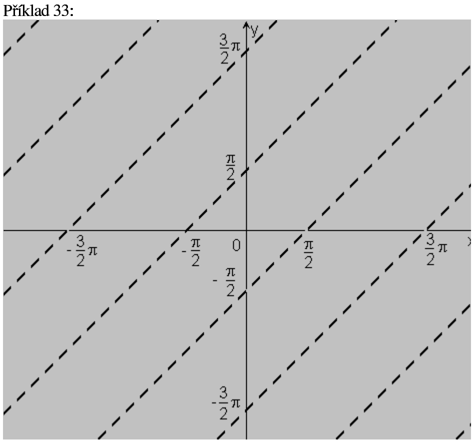


33. $f(x,y) = \tan (x-y)$.

$$x-y \neq (2k+1)\frac{\pi}{2},\ k \in \mathbb{Z} \quad \Rightarrow \quad y \neq x-(2k+1)\frac{\pi}{2}$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \neq x-(2k+1)\frac{\pi}{2},\ k \in \mathbb{Z}\}.$$

Příklad 33:



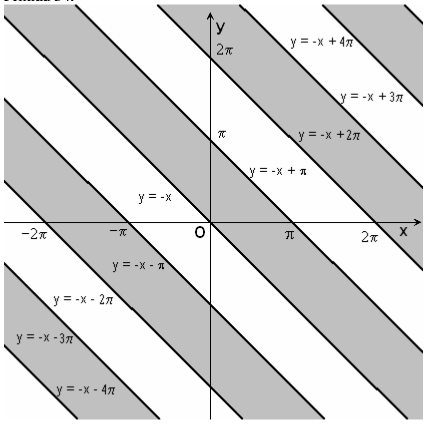
34. $f(x,y) = \sqrt{\sin (x+y)}$.

$$\sin (x+y) \geq 0 \quad \Rightarrow \quad k\pi \leq x+y \leq (k+1)\pi,\ k \in \mathbb{Z}$$

$$\Rightarrow \quad -x+k\pi \leq y \leq -x+(k+1)\pi$$

$$D_f = \{(x,y) \in \mathbb{R}^2; -x+k\pi \leq y \leq -x+(k+1)\pi, k \in \mathbb{Z}\}.$$

Příklad 34:



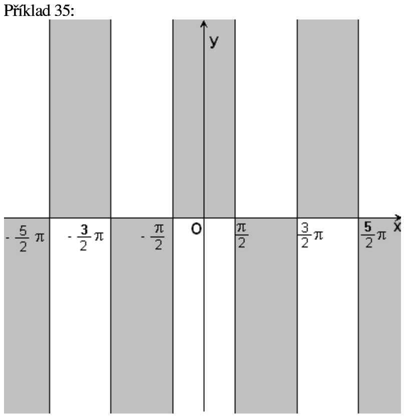
35. $f(x,y) = \sqrt{y \cdot \cos x}$.

$$y \cdot \cos x \geq 0$$

$$(y \geq 0 \wedge \cos x \geq 0) \quad \vee \quad (y \leq 0 \wedge \cos x \leq 0)$$

1. $y \geq 0 \wedge \cos x \geq 0 \Rightarrow y \geq 0 \wedge -\frac{\pi}{2} + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi$
2. $y \leq 0 \wedge \cos x \leq 0 \Rightarrow \frac{\pi}{2} + 2k\pi \leq x \leq \frac{3}{2}\pi + 2k\pi, k \in \mathbb{Z}$.

Příklad 35:



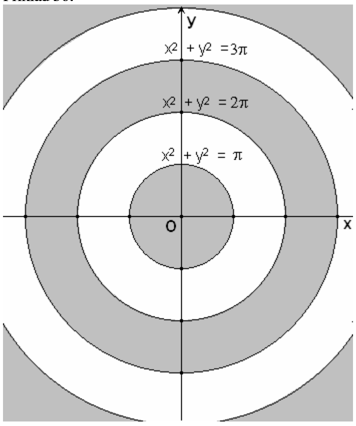
36. $f(x,y) = \sqrt{\sin(x^2+y^2)}$.

$$\sin(x^2+y^2) \geq 0$$

$$\Rightarrow 0 + 2k\pi \leq x^2+y^2 \leq \pi + 2k\pi, k \in \mathbb{Z}^+.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; 2k\pi \leq x^2+y^2 \leq \pi+2k\pi, k \in \mathbb{Z}^+\}.$$

Příklad 36:



37. $f(x,y) = \ln\cos(x+y)$.

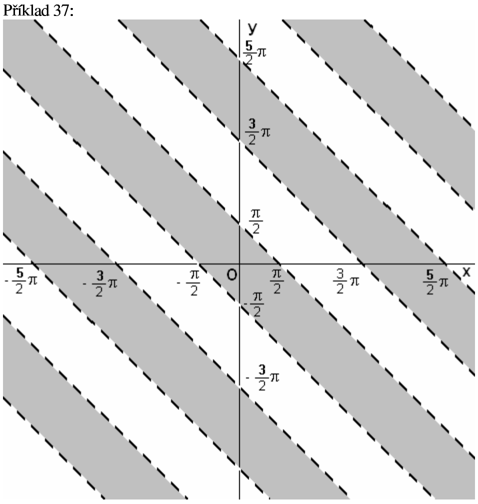
$$\cos(x+y) > 0$$

$$\Rightarrow -\frac{\pi}{2} + 2k\pi < x+y < \frac{\pi}{2} + 2k\pi$$

$$\Rightarrow -x - \frac{\pi}{2} + 2k\pi < y < -x + \frac{\pi}{2} + 2k\pi.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; -x+\frac{\pi}{2}(4k-1) < y < -x+\frac{\pi}{2}(4k+1), k \in \mathbb{Z}\}.$$

Příklad 37:

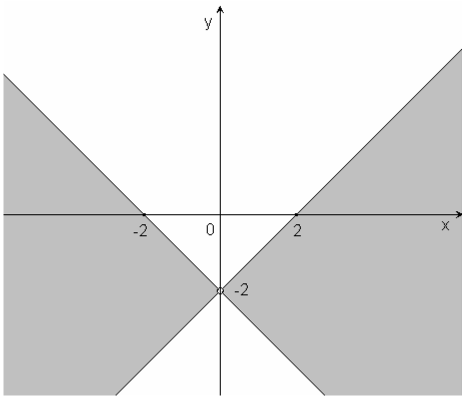


38. $f(x,y) = \arccos\frac{y+2}{x}$.

$$-1 \leq \frac{y+2}{x} \leq 1 \quad \wedge \quad x \neq 0$$

Pro $x > 0: -x \leq y+2 \leq x \Rightarrow -x-2 \leq y \leq x-2$,
pro $x < 0: -x \geq y+2 \geq x \Rightarrow -x-2 \geq y \geq x-2$.

klad 38:



39. $f(x,y) = \arcsin(1-x) + \arccos\frac{y}{x^2}$.

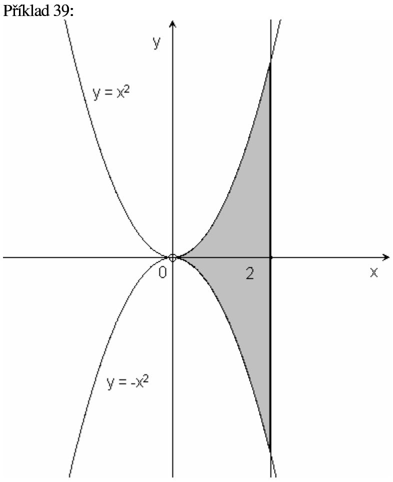
$$-1 \leq 1-x \leq 1 \quad \wedge \quad -1 \leq \frac{y}{x^2} \leq 1 \quad \wedge \quad x \neq 0$$

$$-2 \leq -x \leq 0 \quad \wedge \quad -x^2 \leq y \leq x^2 \quad \wedge \quad x \neq 0$$

$$2 \geq x \geq 0 \quad \wedge \quad -x^2 \leq y \leq x^2 \quad \wedge \quad x \neq 0.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; 0 < x \leq 2 \wedge -x^2 \leq y \leq x^2\}.$$

Příklad 39:



40. $f(x,y) = \arctan \frac{x}{x^2+y}$.

$$x^2 + y \neq 0 \Rightarrow y \neq -x^2.$$

$$D_f = \{(x,y) \in \mathbb{R}^2; y \neq -x^2\}.$$

Příklad 40: