Vypočtěte derivace následujících funkcí:

1. $f(x) = 2x^5 - 7x^4 + 6x - 9$
   $f'(x) = 2 \cdot 5 \cdot x^{5-1} - 7 \cdot 4 \cdot x^{4-1} + 6 \cdot 1 \cdot x^{1-1} - 0 = 10x^4 - 28x^3 + 6$

2. $f(x) = \frac{3}{2x^4} - 4\sqrt[5]{x^2} + \frac{1}{3x} = \frac{3}{2}x^{-4} - 4x^{\frac{2}{5}} + \frac{1}{3}x^{-1}$
$$\begin{aligned} f'(x) &= \frac{3}{2} \cdot (-4) \cdot x^{-4-1} - 4 \cdot \frac{2}{5} \cdot x^{\frac{2}{5}-1} + \frac{1}{3} \cdot (-1) \cdot x^{-1-1} \\ &= -6 \cdot x^{-5} - \frac{8}{5} \cdot x^{-\frac{3}{5}} - \frac{1}{3} \cdot x^{-2} \\ &= -\frac{6}{x^5} - \frac{8}{5\sqrt[5]{x^3}} - \frac{1}{3x^2}. \end{aligned}$$

3. $f(x) = \sqrt{x}(x^5 - 3\sqrt{x} + 2) = x^{\frac{11}{2}} - 3x + 2x^{\frac{1}{2}}$
   $f'(x) = \dfrac{11}{2} \cdot x^{\frac{9}{2}} - 3 + 2 \cdot \dfrac{1}{2} \cdot x^{-\frac{1}{2}} = \dfrac{11}{2}\sqrt{x^9} - 3 + \dfrac{1}{\sqrt{x}}.$

4. $f(x) = \frac{6x^4 - 2\sqrt[3]{x} + 5}{\sqrt{x^3}} = 6x^{\frac{5}{2}} - 2x^{-\frac{7}{6}} + 5x^{-\frac{3}{2}}$
   $f'(x) = 6 \cdot \frac{5}{2}x^{\frac{3}{2}} - 2 \cdot (-\frac{7}{6}) \cdot x^{-\frac{13}{6}} + 5 \cdot (-\frac{3}{2}) \cdot x^{-\frac{5}{2}} = 15x^{\frac{3}{2}} + \frac{7}{3}x^{-\frac{13}{6}} - \frac{15}{2}x^{-\frac{5}{2}} = 15\sqrt{x^3} + \frac{7}{3\sqrt[6]{x^{13}}} - \frac{15}{2\sqrt{x^5}}.$

5. $f(x) = \frac{2x-3}{x^2+4}$
   $f'(x) = \frac{(2x-3)' \cdot (x^2+4) - (2x-3) \cdot (x^2+4)'}{(x^2+4)^2} = \frac{2 \cdot (x^2+4) - (2x-3) \cdot 2x}{(x^2+4)^2} = \frac{-2x^2+6x+8}{(x^2+4)^2} = \frac{-2(x^2-3x-4)}{(x^2+4)^2}.$

6. $f(x) = \frac{x - \sqrt{x}}{x + \sqrt{x}} = \frac{x - x^{\frac{1}{2}}}{x + x^{\frac{1}{2}}}$
   $f'(x) = \frac{(x - x^{\frac{1}{2}})' \cdot (x + x^{\frac{1}{2}}) - (x - x^{\frac{1}{2}}) \cdot (x + x^{\frac{1}{2}})'}{(x + x^{\frac{1}{2}})^2} = \frac{(1 - \frac{1}{2} \cdot x^{-\frac{1}{2}}) \cdot (x + x^{\frac{1}{2}}) - (x - x^{\frac{1}{2}}) \cdot (1 + \frac{1}{2} \cdot x^{-\frac{1}{2}})}{(x + x^{\frac{1}{2}})^2} = \frac{(x - \frac{1}{2}x^{\frac{1}{2}} + x^{\frac{1}{2}} - \frac{1}{2}) - (x - x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2})}{(x + x^{\frac{1}{2}})^2} = \frac{x^{\frac{1}{2}}}{(x + x^{\frac{1}{2}})^2} = \frac{\sqrt{x}}{(x + \sqrt{x})^2}.$

7. $f(x) = \sin x - \cot x$
   $f'(x) = (\sin x)' - (\cot x)' = \cos x - \left(-\frac{1}{\sin^2 x}\right) = \cos x + \frac{1}{\sin^2 x}.$

8. $f(x) = x^3 \cos x$
   $f'(x) = (x^3)' \cdot \cos x + x^3 \cdot (\cos x)' = 3x^2 \cos x + x^3 \cdot (-\sin x) = x^2(3\cos x - x \sin x).$

9. $f(x) = \frac{\tan x}{\sqrt{x}}$
   $f'(x) = \frac{(\tan x)' \cdot \sqrt{x} - \tan x \cdot (\sqrt{x})'}{(\sqrt{x})^2} = \frac{\frac{1}{\cos^2 x} \cdot \sqrt{x} - \tan x \cdot (-\frac{1}{2 \cdot \sqrt{x}})}{x} = \frac{\frac{1}{\cos^2 x} \cdot \sqrt{x} + \frac{\sin x}{\cos x} \cdot \frac{1}{2\sqrt{x}}}{x} = \frac{\frac{2x + \sin x \cos x}{2\sqrt{x}\cos^2 x}}{x} = \frac{2x + \sin x \cos x}{2\sqrt{x^3}\cos^2 x}.$

10. $f(x) = 2^x - e^x + \left(\frac{3}{4}\right)^x$
    $f'(x) = 2^x \cdot \ln 2 - e^x + \left(\frac{3}{4}\right)^x \cdot \ln \frac{3}{4}.$

11. $f(x) = x^2 5^x$
    $f'(x) = (x^2)' \cdot 5^x + x^2 \cdot (5^x)' = 2x \cdot 5^x + x^2 \cdot 5^x \ln 5.$

12. $f(x) = x^3 e^x (\sin x - \cos x)$
    $f'(x) = (x^3)' \cdot e^x \cdot (\sin x - \cos x) + x^3 \cdot (e^x)' \cdot (\sin x - \cos x) + x^3 \cdot e^x \cdot (\sin x - \cos x)' =$
    $3x^2 e^x (\sin x - \cos x) + x^3 e^x (\sin x - \cos x) + x^3 e^x (\cos x + \sin x) = x^2 e^x (3\sin x - 3\cos x + 2x \sin x).$

13. $f(x) = x \ln x$
$f'(x) = (x)' \cdot \ln x + x \cdot (\ln x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1.$

14. $f(x) = \sin x \cdot \log_4 x$
$f'(x) = (\sin x)' \cdot \log_4 x + \sin x \cdot (\log_4 x)' = \cos x \cdot \log_4 x + \sin x \cdot \frac{1}{x} \cdot \frac{1}{\ln 4} = \cos x \cdot \log_4 x + \frac{\sin x}{x \cdot \ln 4}.$

15. $f(x) = \frac{\ln x}{x^5}$
$f'(x) = \frac{(\ln x)' \cdot x^5 - \ln x \cdot (x^5)'}{(x^5)^2} = \frac{\frac{1}{x} \cdot x^5 - \ln x \cdot 5x^4}{x^{10}} = \frac{x^4 - 5x^4 \ln x}{x^{10}} = \frac{1 - 5\ln x}{x^6}.$

16. $f(x) = \sqrt{x} + \arccos x$
$f'(x) = \frac{1}{2 \cdot \sqrt{x}} - \frac{1}{\sqrt{1-x^2}}.$

17. $f(x) = \frac{\arcsin x}{x^2}$
$f'(x) = \frac{(\arcsin x)' \cdot x^2 - \arcsin x \cdot \left(x^2\right)'}{x^4} = \frac{\frac{1}{\sqrt{1-x^2}} \cdot x^2 - \arcsin x \cdot 2x}{x^4} = \frac{\frac{x}{\sqrt{1-x^2}} - 2\arcsin x}{x^3} = \frac{x - 2\sqrt{1-x^2} \cdot \arcsin x}{x^3\sqrt{1-x^2}}.$

18. $f(x) = e^x \arctan x$
$f'(x) = (e^x)' \cdot \arctan x + e^x \cdot (\arctan x)' = e^x \cdot \arctan x + e^x \cdot \frac{1}{1+x^2} = e^x \left(\arctan x + \frac{1}{1+x^2}\right).$

19. $f(x) = (2x^3 + x - 8)^4$
$f'(x) = 4 \cdot (2x^3 + x - 8)^3 \cdot (2x^3 + x - 8)' = 4 \cdot (2x^3 + x - 8)^3 (6x^2 + 1).$

20. $f(x) = \sqrt{x^2 - 3x + 4} = \left(x^2 - 3x + 4\right)^{\frac{1}{2}}$
$f'(x) = \frac{1}{2} \cdot \left(x^2 - 3x + 4\right)^{-\frac{1}{2}} \cdot \left(x^2 - 3x + 4\right)' = \frac{1}{2} \cdot \left(x^2 - 3x + 4\right)^{-\frac{1}{2}} \cdot (2x - 3) = \frac{2x-3}{2 \cdot \sqrt{x^2 - 3x + 4}}.$

21. $f(x) = \frac{3}{\sqrt[3]{7-x^2}} = 3 \cdot \left(7 - x^2\right)^{-\frac{1}{3}}$
$f'(x) = 3 \cdot \left(-\frac{1}{3}\right) \cdot \left(7 - x^2\right)^{-\frac{4}{3}} \cdot (7 - x^2)' = 3 \cdot \left(-\frac{1}{3}\right) \cdot \left(7 - x^2\right)^{-\frac{4}{3}} \cdot (-2x) = \frac{2x}{\sqrt[3]{(7-x^2)^4}}.$

22. $f(x) = \left(x^2 + 1\right) \cdot \sqrt{x^2 - 4x + 11}$
$f'(x) = \left(x^2 + 1\right)' \cdot \sqrt{x^2 - 4x + 11} + \left(x^2 + 1\right) \cdot \left(\sqrt{x^2 - 4x + 11}\right)' = 2x \cdot \sqrt{x^2 - 4x + 11} + \left(x^2 + 1\right) \cdot \frac{1}{2\sqrt{x^2-4x+11}} \cdot \left(x^2 - 4x + 11\right)' = 2x \cdot \sqrt{x^2 - 4x + 11} + \left(x^2 + 1\right) \cdot \frac{1}{2\sqrt{x^2-4x+11}} \cdot (2x - 4) = \frac{2x\left(x^2-4x+11\right) + \left(x^2+1\right)(2x-4)}{2\sqrt{x^2-4x+11}} = \frac{2x^3 - 8x^2 + 22x + 2x^3 - 4x^2 + 2x - 4}{2\sqrt{x^2-4x+11}} = \frac{4x^3 - 12x^2 + 24x - 4}{2\sqrt{x^2-4x+11}}.$

23. $f(x) = \sqrt[4]{\frac{1+x^4}{1-x^4}} = \left(\frac{1+x^4}{1-x^4}\right)^{\frac{1}{4}}$
$f'(x) = \frac{1}{4} \cdot \left(\frac{1+x^4}{1-x^4}\right)^{-\frac{3}{4}} \cdot \left(\frac{1+x^4}{1-x^4}\right)' = \frac{1}{4} \cdot \left(\frac{1+x^4}{1-x^4}\right)^{-\frac{3}{4}} \cdot \frac{4x^3 \cdot (1-x^4) - (1+x^4) \cdot (-4x^3)}{(1-x^4)^2} = \frac{1}{4} \cdot \left(\frac{1-x^4}{1+x^4}\right)^{\frac{3}{4}} \cdot \frac{4x^3 - 4x^7 + 4x^3 + 4x^7}{(1-x^4)^2} = \frac{2x^3}{(1+x^4)^{\frac{3}{4}}(1-x^4)^{\frac{5}{4}}}.$

24. $f(x) = \sin^3 x$
$f'(x) = 3 \cdot \sin^2 x \cdot (\sin x)' = 3\sin^2 x \cos x.$

25. $f(x) = \sin 3x$
$f'(x) = \cos 3x \cdot (3x)' = (\cos 3x) \cdot 3 = 3\cos 3x.$

26. $f(x) = \sin(x^3)$
$f'(x) = \cos(x^3) \cdot (x^3)' = \cos(x^3) \cdot 3x^2 = 3x^2 \cos(x^3).$

27. $f(x) = \sin^5(2x)$
$f'(x) = 5{\cdot}\sin^4(2x){\cdot}(\sin(2x))' = 5{\cdot}\sin^4(2x){\cdot}\cos(2x){\cdot}2 = 10\sin^4(2x){\cdot}\cos(2x).$

28. $f(x) = \sin^2\left(\frac{1-x^3}{1+x^3}\right)$
$f'(x) = 2 \cdot \sin\frac{1-x^3}{1+x^3} \cdot \left(\sin\frac{1-x^3}{1+x^3}\right)' = 2 \cdot \sin\frac{1-x^3}{1+x^3} \cdot \cos\frac{1-x^3}{1+x^3} \cdot \left(\frac{1-x^3}{1+x^3}\right)'$
$= 2 \cdot \sin\frac{1-x^3}{1+x^3} \cdot \cos\frac{1-x^3}{1+x^3} \cdot \frac{(1-x^3)' \cdot (1+x^3) - (1-x^3) \cdot (1+x^3)'}{(1+x^3)^2}$
$= \left| 2 \cdot \sin\frac{1-x^3}{1+x^3} \cdot \cos\frac{1-x^3}{1+x^3} = \sin\left(2 \cdot \frac{1-x^3}{1+x^3}\right) \quad \text{podle vzorce} \quad \sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha \right|$
$= \sin\left(2\frac{1-x^3}{1+x^3}\right) \cdot \frac{-3x^2 \cdot (1+x^3) - (1-x^3) \cdot 3x^2}{(1+x^3)^2}$
$= \frac{-6x^2}{(1+x^3)^2} \cdot \sin\frac{2 \cdot \left(1-x^3\right)}{1+x^3}.$

29. $f(x) = \frac{1}{\tan x} - \sqrt{\cos x} = \tan^{-1} x - \cos^{\frac{1}{2}} x$
$f'(x) = -1 \cdot \tan^{-2} x \cdot (\tan x)' - \frac{1}{2} \cdot \cos^{-\frac{1}{2}} x \cdot (\cos x)'$
$= -\frac{1}{\tan^2 x} \cdot \frac{1}{\cos^2 x} - \frac{1}{2} \cdot \frac{1}{\sqrt{\cos x}} \cdot (-\sin x) = -\frac{1}{\left(\frac{\sin x}{\cos x}\right)^2} \cdot \frac{1}{\cos^2 x} + \frac{\sin x}{2 \cdot \sqrt{\cos x}} =$
$-\frac{1}{\sin^2 x} + \frac{\sin x}{2 \cdot \sqrt{\cos x}}.$

30. $f(x) = x^2 \cdot \sqrt[3]{1 + \cos^4 x} = x^2 \cdot \left(1 + \cos^4 x\right)^{\frac{1}{3}}$
$f'(x) = (x^2)' \cdot \left(1 + \cos^4 x\right)^{\frac{1}{3}} + x^2 \cdot \left(\left(1 + \cos^4 x\right)^{\frac{1}{3}}\right)' =$
$2x \cdot \left(1 + \cos^4 x\right)^{\frac{1}{3}} + x^2 \cdot \frac{1}{3} \cdot \left(1 + \cos^4 x\right)^{-\frac{2}{3}} \cdot \left(1 + \cos^4 x\right)' =$
$2x{\cdot}\left(1 + \cos^4 x\right)^{\frac{1}{3}} + x^2{\cdot}\frac{1}{3}{\cdot}\left(1 + \cos^4 x\right)^{-\frac{2}{3}}{\cdot}4{\cdot}\cos^3 x{\cdot}(-\sin x) = \frac{6x\left(1+\cos^4 x\right) - 4x^2 \sin x \cos^3 x}{3(1+\cos^4 x)^{\frac{2}{3}}}.$

31. $f(x) = \tan(\ln x)$
$f'(x) = \frac{1}{\cos^2(\ln x)} \cdot (\ln x)' = \frac{1}{\cos^2(\ln x)} \cdot \frac{1}{x} = \frac{1}{x\cos^2(\ln x)}.$

32. $f(x) = \ln(\tan x)$
$f'(x) = \frac{1}{\tan x} \cdot (\tan x)' = \frac{1}{\sin x \cdot \cos x} = \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x} = \frac{2}{\sin 2x}.$

33. $\frac{1}{\ln(\sin^2 x)} = \left[\ln\left(\sin^2 x\right)\right]^{-1}$
$f'(x) = (-1) \cdot \left[\ln\left(\sin^2 x\right)\right]^{-2} \cdot \left[\ln\left(\sin^2 x\right)\right]' = \frac{-1}{\ln^2(\sin^2 x)} \cdot \frac{1}{\sin^2 x} \cdot (\sin^2 x)'$
$= \frac{-1}{\ln^2(\sin^2 x)} \cdot \frac{1}{\sin^2 x} \cdot 2 \cdot \sin x \cdot \cos x = \frac{-2\tan x}{\ln^2(\sin^2 x)}.$

34. $f(x) = \cot\frac{1}{1+x^2}$
$f'(x) = -\frac{1}{\sin^2\left(\frac{1}{1+x^2}\right)} \cdot \left(\frac{1}{1+x^2}\right)' = -\frac{1}{\sin^2\left(\frac{1}{1+x^2}\right)} \cdot \frac{(-1)}{(1+x^2)^2} \cdot (x^2)'$
$= -\frac{1}{\sin^2\left(\frac{1}{1+x^2}\right)} \cdot \frac{(-1)}{(1+x^2)^2} \cdot 2x = \frac{2x}{(1+x^2)^2} \cdot \frac{1}{\sin^2\left(\frac{1}{1+x^2}\right)}.$

35. $f(x) = 3^{x^2-2x+5}$
$f'(x) = 3^{x^2-2x+5} \cdot \ln 3 \cdot (x^2 - 2x + 5)' = 3^{x^2-2x+5} \cdot \ln 3 \cdot (2x - 2)$
$= 2(x-1) \cdot 3^{x^2-2x+5} \cdot \ln 3.$

36. $f(x) = e^{\frac{1}{x^2}}$
$f'(x) = e^{\frac{1}{x^2}} \cdot \left(\frac{1}{x^2}\right)' = e^{\frac{1}{x^2}} \cdot \frac{-2}{x^3}.$

37. $f(x) = 5^{\frac{\sqrt{x}}{\ln x}}$
$f'(x) = 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5 \cdot \left(\frac{\sqrt{x}}{\ln x}\right)' = 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5 \cdot \frac{\left(\sqrt{x}\right)' \cdot \ln x - \sqrt{x} \cdot (\ln x)'}{\ln^2 x}$
$= 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5 \cdot \frac{\frac{1}{2\cdot\sqrt{x}} \cdot \ln x - \sqrt{x} \cdot \frac{1}{x}}{\ln^2 x} = 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5 \cdot \frac{\frac{1}{2\cdot\sqrt{x}} \cdot \ln x - \frac{1}{\sqrt{x}}}{\ln^2 x}$
$= 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5 \cdot \frac{\frac{\ln x - 2}{2\cdot\sqrt{x}}}{\ln^2 x} = \frac{\ln x - 2}{2\sqrt{x}\ln^2 x} \cdot 5^{\frac{\sqrt{x}}{\ln x}} \cdot \ln 5.$

38. $f(x) = \ln(7x^3 - 4x^2 + 2x - 5)$
$f'(x) = \frac{1}{7x^3-4x^2+2x-5} \cdot (7x^3-4x^2+2x-5)' = \frac{1}{7x^3-4x^2+2x-5} \cdot (21x^2-8x+2).$

39. $f(x) = \log_5(1 - x^4)$
$f'(x) = \frac{1}{1-x^4} \cdot \frac{1}{\ln 5} \cdot (1 - x^4)' = \frac{1}{1-x^4} \cdot \frac{1}{\ln 5} \cdot (-4x^3) = \frac{-4x^3}{(1-x^4)\cdot\ln 5}.$

40. $f(x) = \ln\sqrt{\frac{1-x^2}{1+x^2}}$
$f'(x) = \frac{1}{\sqrt{\frac{1-x^2}{1+x^2}}} \cdot \left(\sqrt{\frac{1-x^2}{1+x^2}}\right)' = \sqrt{\frac{1+x^2}{1-x^2}} \cdot \frac{1}{2\cdot\sqrt{\frac{1-x^2}{1+x^2}}} \cdot \left(\frac{1-x^2}{1+x^2}\right)'$
$= \frac{1}{2} \cdot \sqrt{\frac{1+x^2}{1-x^2}} \cdot \sqrt{\frac{1+x^2}{1-x^2}} \cdot \frac{(1-x^2)'\cdot(1+x^2)-(1-x^2)\cdot(1+x^2)'}{(1+x^2)^2} = \frac{1}{2} \cdot \frac{1+x^2}{1-x^2} \cdot \frac{-2x\cdot(1+x^2)-(1-x^2)\cdot 2x}{(1+x^2)^2}$
$= \frac{1}{2} \cdot \frac{1}{1-x^2} \cdot \frac{-4x}{1+x^2} = \frac{-2x}{(1-x^2)\cdot(1+x^2)} = \frac{-2x}{1-x^4}.$

41. $f(x) = \arcsin\frac{x-2}{5}$
$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x-2}{5}\right)^2}} \cdot \left(\frac{x-2}{5}\right)' = \frac{1}{\sqrt{1-\frac{x^2-4x+4}{25}}} \cdot \frac{1}{5} = \frac{1}{\sqrt{\frac{21+4x-x^2}{25}}} \cdot \frac{1}{5} = \frac{1}{\sqrt{21+4x-x^2}}.$

42. $f(x) = \arccos\frac{1-x^2}{1+x^2}$
$f'(x) = -\frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \left(\frac{1-x^2}{1+x^2}\right)' = -\frac{1}{\sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{-2x\cdot(1+x^2)-(1-x^2)\cdot 2x}{(1+x^2)^2}$
$= -\frac{1}{\frac{\sqrt{(1+2x^2+x^4)-(1-2x^2+x^4)}}{1+x^2}} = -\frac{1}{\sqrt{4x^2}} \cdot \frac{-2x-2x^3-2x+2x^3}{1+x^2}$
$= -\frac{-4x}{(1+x^2)\cdot 2\cdot|x|} = \frac{2x}{(1+x^2)\cdot|x|} = \frac{2\cdot\operatorname{sgn} x}{1+x^2}.$

43. $f(x) = \frac{1}{\sqrt{5}} \cdot \arctan\frac{x\sqrt{5}}{1-x^2}$
$f'(x) = \frac{1}{\sqrt{5}} \cdot \frac{1}{1+\left(\frac{x\sqrt{5}}{1-x^2}\right)^2} \cdot \left(\frac{x\sqrt{5}}{1-x^2}\right)' = \frac{1}{\sqrt{5}} \cdot \frac{1}{1+\frac{5x^2}{(1-x^2)^2}} \cdot \sqrt{5} \cdot \left(\frac{x}{1-x^2}\right)'$
$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\frac{1-2x^2+x^4+5x^2}{(1-x^2)^2}} \cdot \sqrt{5} \cdot \frac{1\cdot(1-x^2)-x\cdot(-2x)}{(1-x^2)^2} = \frac{1}{x^4+3x^2+1} \cdot \frac{1-x^2+2x^2}{1} = \frac{1+x^2}{x^4+3x^2+1}.$

44. $f(x) = \sqrt[3]{\ln\left(\cos\frac{2x+1}{4}\right)} = \left[\ln\left(\cos\frac{2x+1}{4}\right)\right]^{\frac{1}{3}}$
$f'(x) = \frac{1}{3} \cdot \left[\ln\left(\cos\frac{2x+1}{4}\right)\right]^{-\frac{2}{3}} \cdot \left[\ln\left(\cos\frac{2x+1}{4}\right)\right]'$
$= \frac{1}{3} \cdot \left[\ln\left(\cos\frac{2x+1}{4}\right)\right]^{-\frac{2}{3}} \cdot \frac{1}{\cos\frac{2x+1}{4}} \cdot \left(\cos\frac{2x+1}{4}\right)'$
$= \frac{1}{3} \cdot \left(\ln\cos\frac{2x+1}{4}\right)^{-\frac{2}{3}} \cdot \frac{1}{\cos\frac{2x+1}{4}} \cdot \left(-\sin\frac{2x+1}{4}\right) \cdot \left(\frac{2x+1}{4}\right)'$
$= \frac{1}{3} \cdot \left[\ln\left(\cos\frac{2x+1}{4}\right)\right]^{-\frac{2}{3}} \cdot \left(-\tan\frac{2x+1}{4}\right) \cdot \frac{1}{4} \cdot 2$
$= -\frac{1}{6} \cdot \frac{1}{\sqrt[3]{\left(\ln\cos\frac{2x+1}{4}\right)^2}} \cdot \tan\frac{2x+1}{4}.$

45. $f(x) = x^x == e^{\ln x^x} = e^{x\cdot\ln x}$
$f'(x) = e^{x\cdot\ln x} \cdot (x \cdot \ln x)' = e^{x\cdot\ln x} \cdot (x' \cdot \ln x + x \cdot (\ln x)') = e^{x\cdot\ln x} \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) =$
$x^x\left(\ln x + 1\right).$

46. $f(x)=(1+x)^{1-x}=e^{(1-x)\cdot\ln(1+x)}$
$f'(x)=e^{(1-x)\cdot\ln(1+x)}\cdot[(1-x)\cdot\ln(1+x)]'$
$=e^{(1-x)\cdot\ln(1+x)}\cdot\left[(1-x)'\cdot\ln(1+x)+(1-x)\cdot(\ln(1+x))'\right]$
$=e^{(1-x)\cdot\ln(1+x)}\cdot\left(-1\cdot\ln(1+x)+(1-x)\cdot\frac{1}{1+x}\cdot 1\right)$
$=(1+x)^{1-x}\cdot\left(\frac{1-x}{1+x}-\ln(1+x)\right)$
$=(1+x)^{1-x}\cdot\frac{1-x-(1+x)\cdot\ln(1+x)}{1+x}=\frac{1-x-(1+x)\cdot\ln(1+x)}{(1+x)^x}.$

47. $f(x)=(\sin x)^{\cos x}=e^{\cos x\cdot\ln(\sin x)}$
$f'(x)=e^{\cos x\cdot\ln(\sin x)}\cdot[\cos x\cdot\ln(\sin x)]'$
$=(\sin x)^{\cos x}\cdot\left[-\sin x\cdot\ln(\sin x)+\cos x\cdot\frac{1}{\sin x}\cdot\cos x\right]$
$=(\sin x)^{\cos x}\cdot\frac{1}{\sin x}\cdot\left[\cos^2 x-\sin^2 x\cdot\ln(\sin x)\right].$

48. $f(x)=(\tan x)^{\ln x}=e^{\ln x\cdot\ln(\tan x)}$
$f'(x)=e^{\ln x\cdot\ln\tan x}\cdot[\ln x\cdot\ln(\tan x)]'=(\tan x)^{\ln x}\cdot\left[\frac{1}{x}\cdot\ln(\tan x)+\ln x\cdot\frac{1}{\tan x}\cdot\frac{1}{\cos^2 x}\right]$
$=(\tan x)^{\ln x}\cdot\left[\frac{1}{x}\cdot\ln(\tan x)+\ln x\cdot\frac{1}{\frac{\sin x}{\cos x}}\cdot\frac{1}{\cos^2 x}\right]=(\tan x)^{\ln x}\cdot\left(\frac{1}{x}\cdot\ln(\tan x)+\frac{\ln x}{\sin x\cdot\cos x}\right).$

49. $f(x)=x^{\arccos x}=e^{\arccos x\cdot\ln x}$
$f'(x)=e^{\arccos x\cdot\ln x}\cdot(\arccos x\cdot\ln x)'=x^{\arccos x}\cdot\left(-\frac{1}{\sqrt{1-x^2}}\cdot\ln x+\arccos x\cdot\frac{1}{x}\right).$

50. $f(x)=(\arcsin x)^x=e^{x\cdot\ln(\arcsin x)}$
$f'(x)=e^{x\cdot\ln(\arcsin x)}\cdot[x\cdot\ln(\arcsin x)]'=(\arcsin x)^x\cdot\left[1\cdot\ln(\arcsin x)+x\cdot\frac{1}{\arcsin x}\cdot\frac{1}{\sqrt{1-x^2}}\right]$
$=(\arcsin x)^x\cdot\left[\ln(\arcsin x)+\frac{x}{\sqrt{1-x^2}\cdot\arcsin x}\right].$