Limity s použitím L'Hospitalova pravidla.

1.

$$\lim_{x\to 1} \frac{x^5-1}{2x^3-x-1} = \left[\frac{1-1}{2-1-1} = \frac{0}{0}\right] = \lim_{x\to 1} \frac{(x^5-1)'}{(2x^3-x-1)'} = \lim_{x\to 1} \frac{5x^4}{6x^2-1} = \frac{5}{6-1} = 1$$

2.

$$\lim_{x\to 0} \frac{x}{2-\sqrt{x+4}} = \left[\frac{0}{2-\sqrt{4}} = \frac{0}{0}\right] = \lim_{x\to 0} \frac{(x)'}{(2-\sqrt{x+4})'} = \lim_{x\to 0} \frac{1}{0-\frac{1}{2\sqrt{x+4}}} = \lim_{x\to 0} \frac{-2\sqrt{x+4}}{1}$$

$$= -2\sqrt{0+4} = -4$$

3.

$$\lim_{x\to 1} \frac{x^2-1}{\ln x} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{(x^2-1)'}{(\ln x)'} = \lim_{x\to 1} \frac{2x}{\frac{1}{x}} = \lim_{x\to 1} 2x^2 = 2$$

4.

$$\lim_{x\to 0} \frac{\ln(1+\sin x)}{\sin 4x} = \lim_{x\to 0} \frac{\frac{1}{1+\sin x}\cdot\cos x}{4\cos 4x} = \frac{\frac{1}{1+0}\cdot 1}{4\cdot 1} = \frac{1}{4}$$

5.

$$\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = \left[\frac{1}{0} - \frac{1}{0} = \infty - \infty\right] = \lim_{x\to 0} \frac{x-\sin x}{x\sin x} = \left[\frac{0-0}{0\cdot 0} = \frac{0}{0}\right] = \lim_{x\to 0} \frac{(x-\sin x)'}{(x\sin x)'}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{1\cdot\sin x + x\cos x} = \left[\frac{1-1}{0+0} = \frac{0}{0}\right] = \lim_{x\to 0} \frac{\sin x}{\cos x + 1\cos x - x\sin x} = \frac{0}{1+1-0} = 0$$

6.

$$\lim_{x\to\frac{\Pi}{2}} (1-\sin x)\cdot\tan x = [(1-1)\cdot\infty = 0\cdot\infty] = \lim_{x\to\frac{\Pi}{2}} \frac{1-\sin x}{\text{cotan}\, x} = \left[\frac{1-1}{0} = \frac{0}{0}\right] = \lim_{x\to\frac{\Pi}{2}} \frac{-\cos x}{-\frac{1}{\sin^2 x}}$$

$$= \lim_{x\to\frac{\Pi}{2}} (\sin^2 x\cdot\cos x) = 1^2\cdot 0 = 0$$

7.

$$\lim_{x\to 0+} \left(x\cdot \mathrm{e}^{\frac{1}{x}}\right) = [0\cdot\infty] = \lim_{x\to 0+} \frac{\mathrm{e}^{\frac{1}{x}}}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0+} \frac{\left(\mathrm{e}^{\frac{1}{x}}\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to 0+} \frac{\mathrm{e}^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$$

$$= \lim_{x\to 0+} \mathrm{e}^{\frac{1}{x}} = \left[\mathrm{e}^{\frac{1}{0}} = \mathrm{e}^{\infty}\right] = \infty$$

8.

$$\lim_{x\to 0-} \left(x\cdot \mathrm{e}^{-\frac{1}{x}}\right) = [0\cdot\infty] = \lim_{x\to 0-} \frac{\mathrm{e}^{-\frac{1}{x}}}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0-} \frac{\left(\mathrm{e}^{-\frac{1}{x}}\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to 0-} \frac{\mathrm{e}^{-\frac{1}{x}}\cdot\left(\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$$

$$= -\lim_{x\to 0-} \mathrm{e}^{-\frac{1}{x}} = \left[-\,\mathrm{e}^{-\frac{1}{-0}} = -\,\mathrm{e}^{\infty}\right] = -\infty$$

9.

$$\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{1}{\ln x}\right) = [\infty-\infty] = \lim_{x\to 1} \frac{\ln x - x + 1}{(x-1)\ln x} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{(\ln x - x + 1)'}{[(x-1)\ln x]'}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x}-1}{\ln x + (x-1)\frac{1}{x}} = \left[\frac{1-1}{0+1-1} = \frac{0}{0}\right] = \lim_{x\to 1} \frac{\left(\frac{1}{x}-1\right)'}{\left(\ln x + (x-1)\frac{1}{x}\right)'} = \lim_{x\to 1} \frac{-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} = -\frac{1}{2}$$

10.

$$\lim_{x\to\frac{\Pi}{4}-}(\tan x)^{\tan 2x} = [1^\infty] = \lim_{x\to\frac{\Pi}{4}-} \mathrm{e}^{\tan 2x\cdot\ln\tan x} = \mathrm{e}^{\lim_{x\to\frac{\Pi}{4}-}\tan 2x\cdot\ln\tan x} = \mathrm{e}^{-1}$$

Protože

$$\lim_{x\to\frac{\Pi}{4}-} \tan 2x\cdot\ln\tan x = [\infty\cdot 0] = \lim_{x\to\frac{\Pi}{4}-}\frac{\ln\tan x}{\frac{1}{\tan 2x}} = \left[\frac{0}{0}\right] = \lim_{x\to\frac{\Pi}{4}-}\frac{\frac{1}{\tan x}\cdot\frac{1}{\cos^2 x}}{-\frac{1}{\tan^2 2x}\cdot\frac{1}{\cos^2 2x}\cdot 2}$$

$$= \lim_{x\to\frac{\Pi}{4}-} -\frac{\tan^2 2x\cdot\cos^2 2x}{2\tan x\cdot\cos^2 x} = \lim_{x\to\frac{\Pi}{4}-} -\frac{\frac{4\sin^2 x\cos^2 x}{\cos^2 2x}\cdot\cos^2 2x}{2\frac{\sin x}{\cos x}\cdot\cos^2 x} = \lim_{x\to\frac{\Pi}{4}-} -2\sin x\cos x$$

$$= -2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} = -1$$